Optimal initial condition of passive tracers for their maximal mixing in finite time

MOHAMMAD FARAZMAND, MIT — The efficiency of fluid flow for mixing passive tracers is often limited by fundamental laws and/or design constraints, such that a perfectly homogeneous mixture cannot be obtained in finite time. Here we address the natural corollary question: Given a fluid flow, what is the optimal initial tracer pattern that leads to the most homogeneous mixture after a prescribed finite time? We show that this optimal initial condition coincides with the right singular vector (corresponding to the smallest singular value) of a suitably truncated Perron-Frobenius (PF) operator. The truncation of the PF operator is made under the assumption that there is a small length-scale threshold under which the tracer blobs are considered, for all practical purposes, completely mixed. We demonstrate our results on two examples: a prototypical model known as the sine flow and a direct numerical simulation of two-dimensional turbulence. Evaluating the optimal initial condition through this framework requires only the position of a dense grid of fluid particles at the final instance and their pre-images at the initial instance of the prescribed time interval. As such, our framework can be readily applied to flows where such data are available through numerical simulations or or experimental measurements.