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Optimal bounds and extremal trajectories for time averages in dynamical systems[1] IAN TOBASCO, Univ. of Michigan, DAVID GOLUSKIN, Univ. of Victoria, CHARLES DOERING, Univ. of Michigan — For systems governed by differential equations it is natural to seek extremal solution trajectories, maximizing or minimizing the long-time average of a given quantity of interest. A priori bounds on optima can be proved by constructing auxiliary functions satisfying certain point-wise inequalities, the verification of which does not require solving the underlying equations. We prove that for any bounded autonomous ODE, the problems of finding extremal trajectories on the one hand and optimal auxiliary functions on the other are strongly dual in the sense of convex duality. As a result, auxiliary functions provide arbitrarily sharp bounds on optimal time averages. Furthermore, nearly optimal auxiliary functions provide volumes in phase space where maximal and nearly maximal trajectories must lie. For polynomial systems, such functions can be constructed by semidefinite programming. We illustrate these ideas using the Lorenz system, producing explicit volumes in phase space where extremal trajectories are guaranteed to reside.

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