Simple Second-Order Finite Differences for Elliptic PDEs with Discontinuous Coefficients and Interfaces

CHUNG-NAN TZOU, SAMUEL STECHMANN, University of Wisconsin, Madison — Many multiphase flow problems require the solution of Poisson’s equation with discontinuous coefficients due to different fluid properties, such as density, in the different phases of the fluid. Here we present a second-order-accurate numerical method for this problem, where the method is based on simple finite difference formulas. The derivation is performed on a Cartesian grid and leads to a symmetric operator, even across the interface, with suitable adjustments of the right-hand side arising in the derivation and accounting for the interface. The right-hand side is then determined using an iterative method. Comparisons with other methods, such as the first-order ghost fluid method and the second-order immersed interface method, will be discussed; for instance, the present method does not require derivatives of jump conditions. This numerical method is mathematically proven to be second-order accurate in one dimension, in which case iterations are not needed. Second-order accuracy is demonstrated via numerical trials in both two and three dimensions.

Chung-Nan Tzou
University of Wisconsin, Madison

Date submitted: 01 Aug 2017

Electronic form version 1.4