Do steady rolls maximize heat transport in truncated models of Rayleigh-Bénard convection?¹ DAVID GOLUSKIN, University of Victoria, CHARLES R. DOERING, University of Michigan, ANUJ KUMAR, University of California, Santa Cruz, MATTHEW OLSON, University of Michigan — In Rayleigh-Bénard convection, steady rolls are the simplest nonlinear states. They exist for all Rayleigh numbers (Ra) above the primary instability but are unstable at large Ra. Heat transport by steady rolls is comparable to that by turbulent convection if the aspect ratio of rolls is varied with Ra to maximize heat transport (Waleffe et al., Phys. Fluids 27, 2015). Here we ask: do steady rolls transport more heat than any other flows, including turbulent or unstable time-periodic flows? Answering this question in the affirmative requires computing exact upper bounds on heat transport, and showing that steady rolls saturate the bounds. Exact bounds for the governing PDEs are beyond current capabilities, so we instead study ODE models of increasing dimension, each derived as a Galerkin truncation of the PDEs governing 2D convection between free-slip plates. For the ODEs we compute sharp upper bounds on heat transport by using polynomial optimization. In particular, upper bounds are provided by solutions to convex optimization problems, whose constraints require certain expressions to be sums of squares of polynomials, and which are solved numerically using semidefinite programming. To find the states that saturate these bounds, we perform numerical bifurcation analysis.

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