Velocity and Thermal Boundary Layer Equations for Turbulent Rayleigh-Bénard Convection

EMILY SC CHING, H.S. LEUNG, Department of Physics, The Chinese University of Hong Kong, LUKAS ZWIRNER, OLGA SHISHKINA, Max Planck Institute for Dynamics and Self-Organization — In turbulent Rayleigh-Bénard convection, the boundary layers (BLs) are non-steady with fluctuations, the time-averaged large-scale circulating velocity vanishes far away from the top and bottom plates, and the motion arises from buoyancy. There is no existing BL theory that successfully incorporates all these physical effects. We have derived a full set of BL equations for both the temperature and velocity fields from the Boussinesq equations for a quasi-two-dimensional flow above a heated plate, accounting for all these physical effects. We use the commonly employed concepts of eddy viscosity and eddy thermal diffusivity to study fluctuations and propose a closure model to relate them to the stream function. This full set of BL equations enables us to obtain the time-averaged velocity and temperature BL profiles, in the form of similarity solutions, for general Prandtl number (Pr) in terms of two parameters $k_1$ and $k_2$ that measure the size of fluctuations. We have demonstrated that with a suitable choice of $k_1$ and $k_2$, our theoretical results are good approximations of both the time-averaged velocity and temperature profiles obtained in direct numerical simulations especially at low Pr, for which theoretical results are challenging to obtain.

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