Swimming organisms and robotic vehicles create long-lived flow disturbances that can in principle be detected and even exploited for tracking and navigation. Experimental evidence suggests that many aquatic organisms, from mate seekers to hungry predators, respond to specific hydrodynamic cues created by their respective preys. However, the exact features that make these flows distinguishable and the sensory measurements and layouts that are needed to detect them remain elusive. Here, we consider the inverse problem of classifying flow patterns from local sensory measurements. Specifically, we train neural networks to classify flow patterns by relying on flow sensors that measure a time history of the local flow signal at the sensor location. We systematically investigate the network performance for distinct types of sensory measurements: vorticity, flow velocities parallel and transverse to the direction of flow propagation, and flow speed. We show that the networks trained using transverse velocity outperform other networks, even when subjected to aggressive data corruption. We then train the network to classify flow patterns from instantaneous (one time) measurements, using a spatially-distributed array of sensors. The networks based on the spatially-distributed sensory arrays exhibit remarkable accuracy in flow classification, even when only a handful of sensors are active. We conclude by commenting on the advantages and limitations of these models for flow detection and classification, and we discuss how these results lay the groundwork for developing combined data-driven and physics-based models for flow sensing using distributed sensory arrays.