Reconstructing the piecewise-smooth solution of ordinary differential equations for Chebyshev-collocation solution with pointwise exponential convergence

SANDEEP SAHA, SUDIPTA RAY, Department of Aerospace Engineering, Indian Institute of Technology — Physical problems with interfacial discontinuity in the solution or material property are characterized by piecewise-smooth solutions. Numerical computation of problems with interfacial discontinuity requires accurate resolution of the interface conditions. For finite-order methods, the problem may be resolved with local corrections near the interface. Application of spectral methods to approximate the piecewise-smooth solution without an accurate implementation of interface conditions, however, results in the Gibbs phenomenon and non-convergent numerical solution. In order to overcome the Gibbs phenomenon, the discontinuous solution is expressed as the sum of a smooth function and a modified Heaviside function at the location of the discontinuity. The unit Heaviside step function is modified with a smooth jump function which exactly satisfies the conditions of discontinuity at the interface. A weak form expansion of the jump function that uses interface conditions upto the first derivative for a second-order ordinary differential equation is proposed. Implementation of a Chebyshev-collocation discretization to problems where the discontinuities in the solution are known in analytic form produces numerical solution that converges exponentially in the maximum norm.

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