The discrete and continuous spectrum of a stratified liquid layer[1]

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— We study the temporal spectrum of perturbations in a layer of viscous, density-stratified liquid of infinite depth with a free surface. In this scenario, the canonical set of discrete eigenfunctions, from normal mode analysis, lack completeness. A vorticity patch located at depths larger than the decay length scales of the discrete modes will have its projection predominantly on the continuous spectrum modes. The existence of this continuous spectrum, and the completeness thereof, is ascertained by carrying out a linearized initial value problem(IVP). We validate these findings, using direct numerical simulations(DNS) conducted using an open-source code(basilisk.fr) for solving the incompressible, Navier-Stokes equations for immiscible phases. Further, by using the same methods, we show the absence of the viscous continuous spectrum and the completeness of discrete spectrum in the liquid layer of finite depth. Qualitatively similar results are obtained even with constant density, in a viscous liquid layer of finite/infinite depth. However, for an inviscid fluid of finite extent, we find that the continuous spectrum is essential for the completeness of eigenfunctions in both stratified/unstratified scenarios.

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