Intermediate Nonlinear Development of a Line-tied g-Mode[1]

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The nonlinear gravitational instability (g-mode) of a line-tied plasma flux tube is a prototypical model for edge localized modes (ELMs) in tokamaks and magnetotail substorms. Earlier theory predicted the explosive nonlinear growth of these modes near marginal stability [S. C. Cowley and M. Artun, Phys. Rep., **283**, 185-211 (1997)]. Recent direct MHD simulations with both a finite-difference code and NIMROD indicate that the mode remains bounded in magnitude throughout, from early to intermediate nonlinear phases [P. Zhu, A. Bhattacharjee, and K. Germaschewski, Phys. Rev. Lett. **96**, 065001 (2006); P. Zhu, C. C. Hegna, and C. R. Sovinec, submitted to Phys. Plasmas (2006)]. The mode grows nonlinearly at a rate near or smaller than the linear growth rate, producing shock-like discontinuities and large sheared flows. To understand these simulation results, a new theoretical framework has been developed. The theory is based on an expansion using two small parameters, $\epsilon \sim |\boldsymbol{\xi}|/L_{\mathrm{eq}} \ll 1$, and $n^{-1} \sim k_\parallel/k_\perp \ll 1$, where $\boldsymbol{\xi}$ denotes the plasma displacement, L_{eq} is the characteristic equilibrium scale, and $k_\parallel$ and $k_\perp$ are the dominant wavenumbers of the perturbation parallel and perpendicular to equilibrium magnetic field lines, respectively. When $\epsilon \sim n^{-1}$, the Cowley-Artun regime is recovered where the plasma is incompressible to the lowest order and the Lagrangian compression is very small [$\nabla_0 \cdot \boldsymbol{\xi} \sim \mathcal{O}(n^{-1})$]. In this regime, the nonlinearities only modify the development of the global mode envelop across field lines whereas the local eigenmode structure along field lines remains intact. The detonation regime where the nonlinear growth of the mode tends to a finite-time singularity is a narrower subset of the Cowley-Artun regime. However, this regime is not generic and breaks down when $\epsilon \gg n^{-1}$. In the intermediate nonlinear phase when $\epsilon \sim n^{-1/2}$, the lowest order Lagrangian compression is significant [$\nabla_0 \cdot \boldsymbol{\xi} \sim \mathcal{O}(1)$]. During this phase, the nonlinearities directly influence the growth of the local eigenmodes, and couple the global mode structure across and along field lines. The corresponding governing equations for this intermediate nonlinear phase are derived. Comparison of the predictions of analytic theory and numerical simulations will be discussed.