Dissipation of Energy, Cross Helicity, and Magnetic Helicity in Ideal MHD

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Johns Hopkins University — The “invariants” of ideal MHD—energy, cross helicity, and magnetic helicity—need not be conserved in the limit of zero viscosity and resistivity if the solution fields become singular. This is observed to occur in MHD turbulence, where the effective dissipation is due to nonlinear cascade of the invariants to small-scales. We study the large-scale balances of the three invariants via a “coarse-graining” approach related to Wilson-Kadanoff renormalization group. The ideal dissipation in this framework is due to “turbulent stress” and “turbulent EMF” generated by eliminated plasma motions below the coarse-graining length. We derive upper bounds on these turbulent contributions to the MHD equations and improve the necessary conditions of [1] for ideal dissipation. In particular, we show that the conditions for turbulent dissipation/forward cascade of magnetic helicity are so severe—infinitesimal 3rd-order moments of the velocity & magnetic fields!—that they are unlikely to ever naturally occur. We also establish local balance equations in space-time of the three invariants, both for measurable “coarsed-grained” variables and also for “bare” fields. On this basis we give physical interpretations of the turbulent cascades, in terms of work concepts for energy and in terms of topological linkage [2] for the two helicities. [1] Caflisch et al. 1997 Comm. Math. Phys. 184, 443-455 [2] Moffatt, H. K. 1969 J. Fluid Mech. 35, 117-129.

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