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Building on the Legacy of John Greene: The Transition to Chaos in Volume-Preserving Maps

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In 1968 John Greene began a thirty year odyssey that lead to a detailed understanding of, and precise computational tool for, the onset of chaos in two degree-of-freedom Hamiltonian systems and area-preserving maps. By all mathematical insight of the day, Greene's residue criterion—based on the stability of periodic orbits with rational winding numbers—should have dramatically failed to detect the breakdown of invariant tori. However, it not only worked, but also led to the discovery of self-similarity and renormalization for Hamiltonian dynamics. The residue criterion can be applied, for example, to toroidal magnetic configurations to accurately determine where tori exist and also to develop strategies for enlarging the volume with good surfaces. How much of Greene's vision can be applied to higher dimensional systems? While progress in the Hamiltonian case has been slow, in this talk I will discuss related phenomena for volume-preserving flows and maps. Again a natural application is to magnetic field configurations; however, magnetic nulls now preclude the use of two-dimensional cross sections. Mathematically, families of tori generically arise though a simple bifurcation that leads to the creation of a spheromak or Hill's vortex-like configuration. These tori can be subsequently destroyed by other bifurcations when there are resonances between the toroidal and poloidal winding numbers. Concepts similar to twist, or rotational transform, can be defined for these systems, and transport properties can be computed using a form of lobe dynamics. However, we do not know, as of yet, if there is a residue criterion.