Theory of incompressible MHD turbulence with scale dependent alignment and cross-helicity  
J.J. PODESTA, LANL, A. BHATTACHARJEE, UNH — Boldyrev’s theory of incompressible MHD turbulence is different from that of Goldreich & Sridhar (1995) and others in that it contains a scale dependent alignment of velocity and magnetic field fluctuations in the inertial range. Evidence for this alignment comes from direct numerical simulations and from solar wind data. An extension of Boldyrev’s theory to imbalanced turbulence has been proposed by Perez & Boldyrev. We propose a different theoretical approach which generalizes the results of Perez & Boldyrev and is based on two new solar wind observations. The first is the observation that the normalized cross-helicity is approximately constant in the inertial range. The second is the observation that the probabilities $p$ and $q$ are approximately constant in the inertial range, where $p$ and $q$ are the probabilities that the velocity and magnetic field fluctuations at a randomly chosen point $(x, t)$ are either aligned or anti-aligned, respectively. Aligned means that the angle between the velocity and magnetic field fluctuations is between $0$ and $\pi/2$; anti-aligned means that the angle is between $\pi/2$ and $\pi$. Using these two observational constraints, a generalization of Boldyrev’s theory is constructed in which the cascades of the two Elsasser species are each in a state of critical balance and the eddy geometries are required to be scale-invariant. In the new theory, $E(k_\perp) \propto k_\perp^{-3/2}$, $k_\parallel \propto k_\perp^{1/2}$, and the normalized cross-helicity is scale-invariant (a constant).