Generalized Courant-Snyder theory and Kapchinskij-Vladimirskij distribution for high intensity beams in coupled transverse focusing lattices

HONG QIN, RONALD DAVIDSON, Princeton Plasma Physics Lab, Princeton University, MOSES CHUNG, Fermi National Accelerator Laboratory — Courant-Snyder (CS) theory for uncoupled transverse dynamics of charged particles is generalized to the case of coupled transverse dynamics with two degree of freedom. The generalized theory has the same structure as the original CS theory for one degree of freedom. The four basic components of the original CS theory, i.e., the envelope equation, phase advance, transfer matrix, and the CS invariant, all have their counterparts, with remarkably similarity, in the generalized theory. The envelope function is generalized into an envelope matrix, and the envelope equation becomes a matrix envelope equation with matrix operations that are non-commutative. The generalized theory gives a new parameterization of the 4D symplectic transfer matrix that has the same structure as the parameterization of the 2D symplectic transfer matrix in the original CS theory. For high intensity beams, the generalized Courant-Snyder theory is applied to discover a generalized Kapchinskij-Vladimirskij distribution for high-intensity beams in coupled focusing lattices. This self-consistent distribution, which solves the nonlinear Vlasov-Maxwell equations, is 4D ellipsoid in the phase space. A set of envelope equations are derived to describe the dynamics of the phase space ellipsoid.

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