Stochastic Flux-Freezing for Non-Ideal Hydromagnetic Plasmas

GREGORY EYINK, Applied Math and Statistics, Johns Hopkins University — Non-ideal (viscous and resistive) magnetohydrodynamic plasmas are shown to possess stochastic versions of ideal flux-freezing laws. The magnetic field at a point is equal to the average of an infinite ensemble of field-lines advected to that point by the plasma velocity perturbed with a random white-noise (stochastic Lundquist formula). This implies a stochastic Alfvén theorem, valid for any value of the magnetic Prandtl number. At unit Prandtl number there is also a random version of the generalized Kelvin theorem derived by Bekenstein-Oron for ideal MHD. These stochastic conservation laws are not only consequences of the non-ideal MHD equations, but are in fact equivalent to those equations. Similar results hold for Hall magnetohydrodynamics and the two-fluid plasma model. We argue that flux-conservation remains stochastic for turbulent MHD plasmas in the limit of infinite Reynolds numbers. Infinitely-many field lines are advected to each point by turbulent Richardson diffusion. The reconnection speed for pairs of field lines is the relative velocity of the turbulent fluid at their initial locations. Small-scale turbulent dynamo effect is rigorously related to angular correlation of the individual field vectors before reconnection.

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