Energy stability for symmetric MHD equilibria in the Eulerian description

T. ANDREUSSI, Alta SpA, Pisa, Italy, F. PEGORARO, Pisa University, Pisa, Italy, P.J. MORRISON, The University of Texas at Austin — Magnetized plasma flows with symmetry are analyzed by exploiting the noncanonical Hamiltonian formulation of magnetohydrodynamics (MHD).\(^1\) Because of the symmetry, a flux function is introduced and, by assuming entropy is a flux function, a complete set of Casimir invariants is determined. Translational, axial, and helical symmetries are considered.\(^2\) The energy-Casimir (Eulerian) variational principle for equilibria is analyzed. It is shown that extremal points correspond to equilibria of the symmetric MHD systems, which in the axial symmetric case correspond to the solutions of the generalized Grad-Shafranov equation. For generic equilibria, the second variation of the energy-Casimir functional is presented. A stability matrix is obtained and brought to diagonal form. Positiveness of the second variation is analyzed and a general set of sufficient conditions for stability of the equilibria is deduced. Results are further improved by using a Poincaré inequality, and comparison to the conventional energy principle is made.
