Structural stability of infinite-dimensional Hamiltonian systems\textsuperscript{1}

P.J. MORRISON, IFS, University of Texas at Austin, G.I. HAGSTROM, CIMS, New York University — The stability of physical systems depends on parameter values, especially values where bifurcations to instability occur. Bifurcations for finite-dimensional Hamiltonian systems are detailed by the Krein-Moser theorem, which says instability can only occur through collisions of positive and negative energy modes. Infinite-dimensional Hamiltonian systems differ because of continuous spectra, which complicates the mathematics and the definition of signature essential to the theorem. All ideal plasma theories have Hamiltonian formulations with noncanonical Poisson brackets, which deviate from the canonical case by having a different phase space geometry and this affects the structural stability results we achieved.\textsuperscript{2} For the linearized Vlasov-Poisson (VP) equation we proved that if perturbations are dynamically accessible, bifurcations to instability only occur where the “signature,” $\text{sgn}(u f_0'(u))$, changes, consistent with intuition from finite-dimensions. For non-dynamically accessible perturbations the result fails and the VP system is structurally unstable. The methods used are strong and provided results for other systems, which will be described.

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