A Variational Formulation of Macro-Particle Algorithms for Kinetic Plasma Simulations

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Macro-particle based simulations methods are in widespread use in plasma physics; their computational efficiency and intuitive nature are largely responsible for their longevity. In the main, these algorithms are formulated by approximating the continuous equations of motion. For systems governed by a variational principle (such as collisionless plasmas), approximations of the equations of motion is known to introduce anomalous behavior, especially in system invariants. We present a variational formulation of particle algorithms for plasma simulation based on a reduction of the distribution function onto a finite collection of macro-particles. As in the usual Particle-In-Cell (PIC) formulation, these macro-particles have a definite momentum and are spatially extended. The primary advantage of this approach is the preservation of the link between symmetries and conservation laws. For example, nothing in the reduction introduces explicit time dependence to the system and, therefore, the continuous-time equations of motion exactly conserve energy; thus, these models are free of grid-heating. In addition, the variational formulation allows for constructing models of arbitrary spatial and temporal order. In contrast, the overall accuracy of the usual PIC algorithm is at most second due to the nature of the force interpolation between the gridded field quantities and the (continuous) particle position. Again in contrast to the usual PIC algorithm, here the macro-particle shape is arbitrary; the spatial extent is completely decoupled from both the grid-size and the "smoothness" of the shape; smoother particle shapes are not necessarily larger. For simplicity, we restrict our discussion to one-dimensional, non-relativistic, un-magnetized, electrostatic plasmas. We comment on the extension to the electromagnetic case.

1Supported by the US DoE under contract numbers DE-FG02-08ER55000 and DE-SC0008382.