

DPP14-2014-001741

Abstract for an Invited Paper  
for the DPP14 Meeting of  
the American Physical Society

**Analytical methods for describing charged particle dynamics in general focusing lattices using generalized Courant-Snyder theory<sup>1</sup>**

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The dynamics of charged particles in general linear focusing lattices is analyzed using a generalized Courant-Snyder (CS) theory, which extends the original CS theory for one degree of freedom to higher dimensions. The general focusing lattices are allowed to include quadrupole, skew-quadrupole, solenoidal, and dipole components, as well as variation of beam energy and torsion of the fiducial orbit. The scalar envelope function is generalized into an envelope matrix, and the scalar envelope equation, also known as the Ermakov-Milne-Pinney equation in quantum mechanics, is generalized to an envelope matrix equation. The phase advance is generalized into a 4D symplectic rotation, or an  $U(2)$  element. Other components of the original CS theory, such as the CS invariant, transfer matrix, and Twiss functions all have their counterparts in the generalized theory with remarkably similar expressions. The gauge group of the generalized theory is analyzed. If the gauge freedom is fixed with a desired symmetry, the generalized CS parametrization assumes the form of the modified Iwasawa decomposition, whose importance in phase space quantum mechanics and optics has been recently realized. It is shown that the spectral and structural stability properties of a general focusing lattice are uniquely determined by the generalized phase advance. For structural stability, the generalized CS theory developed enables application of the Krein-Moser theory to significantly simplify the theoretical and numerical analysis. The generalized CS theory provides an effective tool to study the coupled dynamics of high-intensity charged particle beams and to discover more optimized lattice designs in the larger parameter space of general focusing lattices.

<sup>1</sup>Research supported by the U.S. Department of Energy.