Relation between $n=0$ vertical MHD instabilities and neighboring Grad-Shafranov equilibria

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— In tokamaks, the $n = 0$ vertical instability limits the maximum elongation that can be achieved, even with feedback control. Since plasma beta and energy confinement time depend strongly on the elongation, the axisymmetric vertical instability plays a crucial role in overall tokamak performance. We derive an intuitive form of $\delta W_F$, the plasma contribution to the change in total potential energy associated with the $n = 0$ instability. Using this expression, we show that the differential equation for the perturbed flux $\psi$ obtained by setting $\delta W_F = 0$ is identical to the neighboring equilibrium equation obtained by letting $\Psi = \Psi + \psi$ in the Grad-Shafranov equation, and setting the first order contribution to zero. Here $\Psi$ and $\psi$ are the equilibrium and perturbed flux. In other words, $n = 0$ modes at marginal stability can be viewed as neighboring equilibria of the Grad-Shafranov equation. This relationship was long believed to be true, but to the authors’ knowledge, never explicitly derived in the literature.