Conservative discontinuous Galerkin discretizations of the 2D incompressible Euler equation FRANCOIS WAELBROECK, Institute for Fusion Studies, University of Texas at Austin, CRAIG MICHOSKI, Institute for Computational and Engineering Science, University of Texas at Austin, TESS BERNARD, Institute for Fusion Studies, University of Texas at Austin — Discontinuous Galerkin (DG) methods provide local high-order adaptive numerical schemes for the solution of convection-diffusion problems. They combine the advantages of finite element and finite volume methods. In particular, DG methods automatically ensure the conservation of all first-order invariants provided that single-valued fluxes are prescribed at inter-element boundaries. For the 2D incompressible Euler equation, this implies that the discretized fluxes globally obey Gauss’ and Stokes’ laws exactly, and that they conserve total vorticity. Liu and Shu [J. Comp. Phys. 160, 577 (2000)] have shown that combining a continuous Galerkin (CG) solution of Poisson’s equation with a central DG flux for the convection term leads to an algorithm that conserves the principal two quadratic invariants, namely the energy and enstrophy. Here, we present a discretization that applies the DG method to Poisson’s equation as well as to the vorticity equation while maintaining conservation of the quadratic invariants. Using a DG algorithm for Poisson’s equation can be advantageous when solving problems with mixed Dirichlet-Neuman boundary conditions such as for the injection of fluid through a slit (Bickley jet) or during compact toroid injection for tokamak startup.

Francois Waelbroeck
Institute for Fusion Studies, University of Texas at Austin

Date submitted: 13 Jul 2016
Electronic form version 1.4