Extending geometrical optics: A Lagrangian theory for vector waves

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Even diffraction aside, the commonly known equations of geometrical optics (GO) are not entirely accurate. GO considers wave rays as classical particles, which are completely described by their coordinates and momenta, but rays have another degree of freedom, namely, polarization. As a result, wave rays can behave as particles with spin. A well-known example of polarization dynamics is wave-mode conversion, which can be interpreted as rotation of the (classical) “wave spin.” However, there are other less-known manifestations of the wave spin, such as polarization precession and polarization-driven bending of ray trajectories. This talk presents recent advances in extending and reformulating GO as a first-principle Lagrangian theory, whose effective-gauge Hamiltonian governs both mentioned polarization phenomena simultaneously. Examples and numerical results are presented. When applied to classical waves, the theory correctly predicts the polarization-driven divergence of left- and right- polarized electromagnetic waves in isotropic media, such as dielectrics and nonmagnetized plasmas. In the case of particles with spin, the formalism also yields a point-particle Lagrangian model for the Dirac electron, i.e. the relativistic spin-1/2 electron, which includes both the Stern-Gerlach spin potential and the Bargmann-Michel-Telegdi spin precession. Additionally, the same theory contributes, perhaps unexpectedly, to the understanding of ponderomotive effects in both wave and particle dynamics; e.g., the formalism allows to obtain the ponderomotive Hamiltonian for a Dirac electron interacting with an arbitrarily large electromagnetic laser field with spin effects included.

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