Simulating Pitch Angle Scattering Using an Explicitly Solvable Energy Conserving Algorithm\(^1\) YICHEN FU, XIN ZHANG, Princeton University, HONG QIN, Princeton Plasma Physics Laboratory — The Langevin equation, a stochastic differential equation (SDE) equivalent to the Fokker-Plank equation in describing the collisional plasma, is frequently used in particle-based simulations. For the pitch angle scattering defined by the Lorentz operator, the energy of particles is exactly conserved, whereas most SDE algorithms have large long-term energy errors that degrade the convergence of the SDE algorithms. For example, we show that the standard Euler-Maruyama method, whose strong order of convergent is 1/2, does not converge for the pitch angle scattering due to the lack of global Lipschitz condition in the range of solutions. To overcome this difficulty, we design a novel explicitly solvable structure-preserving algorithm for the Langevin equation describing pitch angle scattering in a background electromagnetic field [arXiv:2006.10877]. The proposed algorithm utilizes the Cayley transform to calculate the velocity rotation, which preserves exactly the norm of velocity. Using Ito calculus, we prove that the strong order of convergent of the proposed algorithm is 1/2. The long-time accuracy of the algorithm has also been benchmarked and verified numerically. The algorithm is being applied to study the physics of runaway electrons in tokamaks.

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