The Redmond Formula with Seniority

LARRY ZAMICK, ALBERTO ESCUDEROS, Rutgers U. — As we get to heavier nuclei, we find more states with different seniorities and several states of a given seniority. There is a recursion formula by Redmond that relates an $n \rightarrow (n + 1)$ coefficient of fractional parentage (cfp) to that of $(n - 1) \rightarrow n$. However, this involves an overcomplete set of principal parent (pp) cfp’s. For example, for a 3-particle system, we can form basis states $\begin{pmatrix} 12 \end{pmatrix}^{J_0} 3$, where $J_0$ is the pp; we then antisymmetrize and normalize $\Psi[J_0] = N[J_0](1 - P_{12} - P_{13}) \begin{pmatrix} 12 \end{pmatrix}^{J_0} 3$, and form a ppcfp expansion $\Psi[J_0] = \sum \begin{pmatrix} j^2(J_1) j \end{pmatrix} j^3[J_0] J \begin{pmatrix} 12 \end{pmatrix}^{J_0} 3$. But for, say, $J = j = 9/2$, there are five $\Psi[J_0]$’s, but only two independent wave functions, one with seniority 1 and one with seniority 3. We note that $\begin{pmatrix} j^2(J_0) j \end{pmatrix} j^3[J_0] J = 1/(3N[J_0])$. We are able then to obtain the following relation between overcomplete ppcfp’s and complete orthonormal cfp’s: $A = B = C$, where

$$A = (n + 1)\begin{pmatrix} j^n(J_0 v_0) j \end{pmatrix} J^{n+1}[J_0 v_0] J \begin{pmatrix} j^n(J_1 v_1) j \end{pmatrix} J^{n+1}[J_0 v_0] J,$$

$$B = (n + 1) \sum \begin{pmatrix} j^n(J_0 v_0) j \end{pmatrix} J^{n+1} J v \begin{pmatrix} j^n(J_1 v_1) j \end{pmatrix} J^{n+1} J v,$$

$$C = \delta_{J_0 J_1} \delta_{v_0 v_1} + n(-1)^{J_0 + J_1} \sqrt{(2J_0 + 1)(2J_1 + 1)} \sum_{v_2 J_2} J_2 J_1 J j J_0 \times\begin{pmatrix} j^{n-1}(J_2 v_2) j \end{pmatrix} J^n J_0 v_0 \begin{pmatrix} j^{n-1}(J_2 v_2) j \end{pmatrix} J^n J_1 v_1.$$