Maximum J Pairing and Asymptotic Behavior of the 3j and 9j Coefficients

DANIEL HERTZ-KINTISH, LARRY ZAMICK, BRIAN KLESZYK, Rutgers University — We investigate the large \( j \) behavior of certain 3\( j \) and 9\( j \) symbols, where \( j \) is the total angular momentum of one particle in a given shell. Our motivation is the problem of maximum \( J \) pairing in nuclei, along with the more familiar \( J = 0 \) pairing. Maximum \( J \) pairing leads to an increase in \( J = 2 \) coupling of two protons and two neutrons relative to \( J = 0 \). We find that a coupling unitary 9\( j \) symbol \( (U9j) \) is very weak as \( j \) increases, leading to wavefunctions which are to an excellent approximation single \( U9j \) coefficients. Our study of the large \( j \) behavior of coupling unitary 9\( j \) symbols is through the consideration of the case when the total angular momentum \( I \) is equal to \( I_{\text{max}} - 2n \) and \( n = 0, 1, 2, \ldots \). We here derive asymptotic approximations of coupling 3\( j \) symbols and find that the 3\( j \) \( \propto j^{-3/4} \) in the high \( j \) limit. One major analytical tool we used is the Stirling Approximation. Through analytical, numerical, and graphical methods, we show the power law behavior of the coupling unitary 9\( j \) symbols in the \( n/j \ll 1 \) limit, i.e. \( U9j \propto j^{-n} \). Power-law behavior is evident if there is a linear dependence of \( \ln |U9j| \) vs. \( \ln j \). We also present some examples of percent errors in our approximations.

1Daniel Hertz-Kintish and Brian Kleszyk thank the Aresty Center for Undergraduate Research for support during the 2014 summer session and the 2013-2014 academic year, respectively.