Extinction Times for Birth-Death Processes: Continuum Asymptotics and the failure of the Fokker-Planck Approximation

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