Chaos Beyond Linearized Stability Analysis: Folding of the Phase Space and Distribution of Lyapunov Exponents

PETER SILVESTROV, Instituut-Lorentz, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands, I.V. PONOMAREV, Naval Research Laboratory, Washington, DC 20375, USA — We have found [?] a universal mechanism leading to the enhanced probability, \( P(\lambda, t) \), to find small values of the finite time Lyapunov exponent, \( \lambda \). In our investigation of chaotic dynamical systems we go beyond the linearized stability analysis of nearby divergent trajectories and consider folding of the phase space in the course of chaotic evolution. We show that the spectrum of the Lyapunov exponents \( F(\lambda) = \lim_{t \to \infty} t^{-1} \ln \ln P(\lambda, t) \) at the origin has a finite value \( F(0) = -\tilde{\lambda} \) and a slope \( F'(0) \leq 1 \). This means that all negative moments of the distribution \( \langle e^{-m\lambda t} \rangle \) are saturated by rare events with \( \lambda \to 0 \). Extensive numerical simulations confirm the results. Among the practical applications of our findings are the problem of a gap in spectrum of a semiclassical Andreev billiard, conductance fluctuations in a smooth quantum dot and stability to perturbations in chaotic wave-packet dynamics.

References