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Positivity Preserving Nonstandard Finite Difference Schemes for PDE's Having Cross-Diffusion Terms[1] RONALD MICKENS, Clark Atlanta University — Many phenomena in the natural and engineering sciences can be modeled by coupled systems of nonlinear partial differential equations (PDE) in which cross-diffusion terms occur. Such terms correspond to expressions for which the diffusion coefficients depend on dependent variables other than the one represented in the first-order time derivative for a given equation. An example is $(vu_x)_x$, where v and u are two different dependent variables. Since such terms can appear as either a positive or negative sign, it becomes critical to construct numerical integration schemes that preserve a positivity condition. The positivity condition is the requirement that if $(v, u, \ldots)$ are non-negative at $t = 0$, then as they evolve in time, they remain non-negative. Since, for many systems the variables $(v, u, \ldots)$ are densities or particle numbers, the significance of a positivity condition is obvious. Since many of the standard numerical integration methods do not strictly enforce this condition, they may give rise to numerical instabilities, i.e., solutions to the discrete equations not corresponding to any actual solutions of the PDE's. We demonstrate that the nonstandard finite difference procedures of Mickens [1] can provide positivity preserving schemes. We are also able to obtain relationships between the space and time step-sizes.
[1] R. E. Mickens, Finite Difference Models of Differential Equations (World Scientific, 1994).

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