Quantized Interest Rate At The-money for American options

LAMINE DIENG, SAMIR LIPOVACA — In this work, we expand the idea of Shepp for stock optimization using the Bachelier model as our model for the stock price at the money \((X = K)\) for the American call and put options. At the money \((X = K)\) for American options, the expected payoff of both the call and put options is zero. Shepp investigated several stochastic optimization problems using martingale and stopping time theories, one of the problems he investigated was how to optimize the stock price using both the Black-Scholes and the Bachelier (additive) models for the American option above the strike price \(K\) (exercise price) to a stopping point. In order to explore the non-relativistic quantum effect on the expected payoff for both the call and put options at the money \((X = K)\), we assumed the stock price to undergo a stochastic process governed by the Bachelier model given above. Further, using Ito calculus and martingale theory, we obtained a differential equation for the expected payoff for both the call and put options in terms delta and gamma. We investigated the solution of the differential equation in the limit when delta is zero, this sometimes is called hedging or delta neutral in finance. By comparison, the delta-hedged differential equation corresponded to the non-relativistic time-independent Schrödinger equation in quantum mechanics with a constant diffusion constant. We solved exactly the non-relativistic Schrödinger equation at the money and obtained a quantized interest rate in terms of volatility and stock price.