Narrow Escape AMIT SINGER, ZEEV SCHUSS, Department of Mathematics, Tel Aviv University, DAVID HOLCMAN, Department of Mathematics, Weizmann Institute of Science, BOB EISENBERG, Department of Molecular Biophysics and Physiology, Rush Medical Center — A Brownian particle is confined in a finite domain (2D or 3D) where it can exit only through a small opening of the boundary. The calculation of the mean exit time is formulated as a mixed Neumann-Dirichlet boundary value problem in potential theory. We find an asymptotic expansion for the mean exit time in terms of the geometry. The singular flux profile through the opening is obtained. When the small opening is located at non-smooth parts of the boundary, such as corners and cusps, the asymptotic expansion takes a different form. The problem arises in several biological applications, such as ion permeation in protein channels (3D) and receptor diffusion on the surface of nerve cells (2D Riemann surface). The rate at which ions arrive to the mouth of a protein channel (or to a microscopic simulation volume) is a geometrical function that involves both microscopic and macroscopic scales. The volume and surface area of the simulation region is microscopic, whereas the volume of the surrounding bath is macroscopic. From the analysis, it becomes clear why ions take so long to enter the channel.

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