Molecular beam epitaxy enables one to synthesize HTS thin films with rms surface roughness in the range 0.2-0.5 nm, much less than the unit cell height (1-2 nm). One can also make atomically smooth multilayers and superlattices in which HTS or spacer layers can be just one unit cell thick if so desired. A detailed study of transport properties of such heterostructures has already revealed some unexpected findings. In junctions where the barrier is made out of underdoped cuprate with a reduced critical temperature $T_c$, we observe the Giant Proximity Effect: supercurrent runs through very thick barrier layers even at temperature well above $T_c$ (contrary to what is expected from the standard theory). Atomic smoothness of films and multilayers, excellent device uniformity, and reversible modulation of barrier properties by oxygen intake provided solid evidence against experimental artifacts such as pinholes and micro-shorts. Hence, the effect is real and intrinsic, and it defies the conventional explanation. Interpretation and significance of our experimental results will be discussed in the context of theoretical concepts such as the pseudogap, midgap states, electronic inhomogeneity, preformed pairs, and possibly resonant pair tunneling. The work at BNL is done in collaboration with G. Logvenov, V. Butko, A. Gozar and A. Bollinger.