Correlation functions and anisotropy in the XY model with a magnetic field

WEI ZHANG*, H.A. FERTIG, Department of Physics, Indiana University — Recent studies have demonstrated that the classical XY model with a magnetic field has three phases: a linearly confined vortex-antivortex phase, a logarithmically (Log) confined phase, and a free vortex phase. We calculate correlation functions in this model by making use of duality transformations and numerical simulations. In all three phases, the order parameter is nonzero and
\[
\langle \cos(\theta(r_1)) \cos(\theta(r_2)) \rangle \rightarrow \text{const.}, \quad \text{for } |r| = |r_1 - r_2| \rightarrow \infty,
\]
indicating true long range order, when the ordering field favors \( \theta = 0 \). A renormalization group analysis suggests that the Log phase may be distinguished from the other two in the correlation function
\[
C(r) = \langle \sin(\theta(r_1)) \sin(\theta(r_2)) \rangle,
\]
which is short-ranged with a correlation length that depends on the direction of \( r_1 - r_2 \) with respect to the lattice axes. However, it is shown that a directional anisotropy also enters all the phases through a prefactor. Our numerical simulations support the existence of these anisotropies, but the the anisotropy in the prefactor is generically relatively strong while the correlation length anisotropy is rather weak, making it difficult to distinguish the different phases from these correlation functions.

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1This work was supported by NSF Grant No. DMR-0454699