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Critical 2-D Percolation: Crossing Probabilities, Modular Forms and Factorization[1]

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We first consider crossing probabilities in critical 2-D percolation in rectangular geometries, derived via conformal field theory. These quantities are shown to exhibit interesting modular behavior [1], although the physical meaning of modular transformations in this context is not clear. We show that in many cases these functions are completely characterized by very simple transformation properties. In particular, Cardy's function for the percolation crossing probability (including the conformal dimension 1/3), follows from a simple modular argument.
We next consider the probability of crossing between various points for percolation in the upper half-plane. For two points, with the point x an edge of the system, the probability is

$$\mathcal{P}(x,z) = k\frac{1}{y^{5/48}}\Phi(x,z)^{1/3} \tag{1}$$

where Φ is the potential at z of a 2-D dipole located at x, and k is a non-universal constant. For three points, one finds the exact and universal factorization [2,3]

$$\mathcal{P}(x_1,x_2,z) = C\,\sqrt{\mathcal{P}(x_1,z)\mathcal{P}(x_2,z)\mathcal{P}(x_1,x_2)} \tag{2}$$

with

$$C = \frac{8\sqrt{2}\,\pi^{5/2}}{3^{3/4}\,\Gamma(1/3)^{9/2}}. \tag{3}$$

These results are calculated by use of conformal field theory. Computer simulations verify them very precisely. Furthermore, simulations show that the same factorization holds asymptotically, with the same value of C, when one or both of the points x_i are moved from the edge into the bulk.

1.) Peter Kleban and Don Zagier, Crossing probabilities and modular forms, J. Stat. Phys. 113, 431-454 (2003) [arXiv: math-ph/0209023].
2.) Peter Kleban, Jacob J. H. Simmons, and Robert M. Ziff, Anchored critical percolation clusters and 2-d electrostatics, Phys. Rev. Letters 97,115702 (2006) [arXiv: cond-mat/0605120].
3.) Jacob J. H. Simmons and Peter Kleban, in preparation.

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