Pair Binding in Small Hubbard Clusters

W.-F. TSAI, Dept. of Physics, UCLA, H. YAO, S. KIVELSON, Dept. of Physics, Stanford Univ., A. LAUCHLI, IRRMA, EPF Lausanne, Switzerland — One of the key issues in high-$T_c$ superconductors is how (and whether) high temperature pairing can arise in an electronic system with only repulsive interactions. Here, we report the results of analytic and numerical exact diagonalization studies of small Hubbard clusters (up to 16 sites). Taking the N-electron ground-state as the “vacuum state” of the cluster, we define the effective interaction between two added electrons to be $V^{\text{eff}}(N) = E(N+2)+E(N)-2E(N+1)$, where $E(N)$ is the ground-state energy with N electrons on the cluster. Not surprisingly, for most clusters and most values of N, $V^{\text{eff}}$ is repulsive ($V^{\text{eff}}>0$), but there exist special clusters in which, for special N and in an appropriate range of $U/t$, there is an effective attraction, $V^{\text{eff}}<0$. In the weak coupling limit ($U/t<<1$), the results can be understood within perturbation theory, and the effective attraction, where it occurs, is associated with the existence of an anomalous “resonantly entangled” groundstate. In the strong coupling limit, $V^{\text{eff}}$ is always positive (or zero) due to Nagaoka physics. In some sense, the optimal cluster is the Hubbard-tetrahedron, for which $V^{\text{eff}}$ is negative for all $U/t$. Finally, by studying the dependence of $V^{\text{eff}}$ on the patterns of inhomogeneous couplings within a single cluster, we obtain some insight into the issue of whether there exists an optimal inhomogeneity for high-$T_c$ superconductivity.