Spectral theory of extreme statistics in birth-death systems

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Statistics of rare events, or large deviations, in chemical reactions and systems of birth-death type have attracted a great deal of interest in many areas of science including cell biochemistry, astrochemistry, epidemiology, population biology, etc. Large deviations become of vital importance when discrete (non-continuum) nature of a population of “particles” (molecules, bacteria, cells, animals or even humans) and stochastic character of interactions can drive the population to extinction. I will briefly review the novel spectral method [1-3] for calculating the extreme statistics of a broad class of birth-death processes and reactions involving a single species. The spectral method combines the probability generating function formalism with the Sturm-Liouville theory of linear differential operators. It involves a controlled perturbative treatment based on a natural large parameter of the problem: the average number of particles/individuals in a stationary or metastable state. For extinction (the first passage) problems the method yields accurate results for the extinction statistics and for the quasi-stationary probability distribution, including the tails, of metastable states. I will demonstrate the power of the method on the example of a branching and annihilation reaction, $A \rightarrow 2A$, $2A \rightarrow \emptyset$, representative of a rather general class of processes.


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