Anomalous size dependence of inverse participation ratio of localized eigenfunctions in graded elastic lattices

M.J. ZHENG, Chinese University of Hong Kong, M. GODA, Niigata University, K. YAKUBO, Hokkaido University, K.W. YU, Chinese University of Hong Kong — Recently, we studied harmonic vibrational excitations in graded elastic lattices [1]. It is found that the eigenfunctions exhibit a transition from extended phonon states to localized gradon states when the frequency is increased beyond a critical frequency called the gradon transition frequency. At the same time, the inverse participation ratio (IPR) can exhibit a rapid increase at gradon transition. This unusual behavior prompts us to study the size dependence of the IPR of gradon wave function. A quantum analogue is established for the hump structure at the gradon front, via the fact that the probability of a quantum particle is inversely proportional to its velocity. In this way, the envelope function can be determined analytically, and matches the gradon wave function quite well. We find that the size dependence can be captured by the relation: \( N \text{IPR} = C_1 \log(N) + C_2 \), where \( C_1 \) and \( C_2 \) are constants. The interpretation is important in the understanding of a wide variety of properties of graded systems.


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