Quantum memory that can protect qubit states against decoherence is an important piece of a scalable quantum computing architecture. Coupling a qubit to a high-Q harmonic oscillator memory element is one example, but many other quantum systems could serve this role. We have used an atomic two-level state (TLS) in the amorphous AlO$_x$ tunnel barrier of a Josephson phase qubit as a prototype quantum memory element. The frequency-tunability of the phase qubit allows us to switch on and off the qubit-TLS coupling and thereby transfer arbitrary qubit states into the TLS, store them for some time and recall them later. We performed quantum process tomography to completely characterize the memory operation and the errors that occur during the state transfer and recall. The overall process fidelity is 78%. The dominant operator-sum errors are dephasing-like ($\sim 12\%$) and relaxation-like ($\sim 9\%$), consistent with the measured $T_1$ and $T_2$ of the TLS.