Physics of a Bose gas in 2D is quite different from the usual 3D situation. In a homogeneous 2D fluid of identical bosons long-range order is always destroyed by long wavelength thermal fluctuations and this system cannot undergo conventional Bose-Einstein condensation. Nevertheless, it can become superfluid at a finite critical temperature. This phase transition does not involve any symmetry breaking and in the Berezinskii-Kosterlitz-Thouless (BKT) paradigm it is explained in terms of binding and unbinding of pairs of vortices with opposite circulations. Above the critical temperature, proliferation of unbound vortices is expected. Using optical lattice potentials we can create two parallel, independent 2D atomic clouds with similar temperatures and chemical potentials. When the clouds are suddenly released from the trapping potential and allowed to freely expand, they overlap and interfere. This realizes a matter wave heterodyning experiment which gives direct access to several features of the phase distributions in the two planes. Long wavelength phase fluctuations create a smooth and random variation of the interference fringes and free vortices appear as sharp dislocations in the interference pattern. Both the temperature study of these effects and the measurements of the critical point support the BKT picture of the development of quasi-long-range coherence in these systems.