Optimality and Directionality in Network Synchronization
TAKASHI NISHIKAWA, Clarkson University, ADILSON MOTTER, Northwestern University — In a network of dynamical elements, one of the most fundamental issues is the relationship between the network structure and the collective dynamics of the system. The study of complete synchronization, a simplest form of collective dynamics in a network, in which all oscillators behave in precisely the same way, provides an excellent starting point for understanding how collective behavior arises in a network. The stability of complete synchronization in a weighted directed network of oscillators can be formulated using the well known master stability function and the eigenvalues of the Laplacian matrix encoding the topological structure of the network. In this talk, I will use this formulation to address an interesting optimization problem: which network topology has the highest synchronizability? I will first show that the optimality condition can be expressed solely in terms of the Laplacian eigenvalues. The class of optimal networks contains all directed trees with appropriate connection weights, and most in the class have well-defined directionality. I will also discuss the robustness of optimality against the structural perturbation, as well as the role of directionality in the connectivity patterns in enhancing the synchronizability.