Chaotic Dynamics of an Elastically Bouncing Dumbbell COLIN REES, SCOTT FRANKLIN, Rochester Institute of Technology — The dynamics of an elastically bouncing dumbbell is analogous to those of a ball bouncing on a sinusoidally oscillating surface with one important exception: the dumbbell’s angular velocity, analogous to the surface’s oscillation frequency, changes with each bounce, making the subsequent motion significantly more complicated. We investigate this dynamical system over a range of aspect ratios and initial energy, finding periodic, quasi-periodic and chaotic motions. As the initial energy is increased, the dumbbell can flip over and tumble. We find for large aspect ratios, however, narrow bands of energies well above this minimum where tumbling suddenly ceases. Because energy is conserved, the dynamics of a bounce are uniquely determined by the angle and angular velocity. The Lyapunov exponents of paths in this two dimensional phase space can be calculated, with the hope of identifying periodic islands within the chaotic sea. Finally, for certain parameters, the angle at each collision moves from its initial value in a subdiffusive manner, and we determine the characteristic exponents.

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