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**The undulating shape of growing ribbons**

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The undulating morphology of leaves and petals is now accepted as a consequence of differential growth of the underlying tissue. Various qualitative and quantitative aspects of the buckling patterns seen in both vascular and avascular leaves may thus be ascribed to the distribution of non-uniform growth in the lamina, and have been demonstrated in normal and mutant leaves, as well as in physical models thereof. To understand the different modalities that arise quantitatively, we construct a mathematical model for the stability of an initially flat or curved elastic ribbon with gradients in growth directly motivated by observations of kelp that are capable of phenotypic plasticity in different environments. Using a combination of analysis, numerical simulation, and experimental observations, we map out the phase space of possible shapes for these growing ribbons. In general, we find that as the relative growth strain is increased, the ribbon-like structure first switches to a catenoidal shape before developing undulating edges that can develop on the catenoid's edges. Our framework allows us to delineate the few macroscopic parameters that control the morphology of elongated leaves and flower petals and helps to explain the large variety of observed shapes.