Theory of Topological Insulators
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Topological insulators are materials with a bulk excitation gap generated by the spin orbit interaction, and which are different from conventional insulators. This distinction is characterized by $\mathbb{Z}_2$ topological invariants, which characterize the ground state. In two dimensions a single $\mathbb{Z}_2$ invariant describes the quantum spin Hall insulator phase. In three dimensions there are four $\mathbb{Z}_2$ invariants, distinguishing “weak” (WTI) and “strong” (STI) topological insulators. The STI phase is characterized by the presence of unique gapless surface states whose Fermi surface encloses an odd number of 2D Dirac points. We will argue theoretically that the semiconducting alloy Bi$_{1-x}$Sb$_x$ is a strong topological insulator – a prediction that has recently been confirmed experimentally. We will next show that the proximity effect between this unique surface phase and an ordinary superconductor leads to a two dimensional state that resembles a spinless $p_x + ip_y$ superconductor, but does not break time reversal symmetry. This state supports zero energy Majorana bound states at vortices, and may provide a new venue to realize proposals for topological quantum computing.