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Lattice Boltzmann approaches to magnetohydrodynamics and electromagnetism[1]

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We present a lattice Boltzmann approach for magnetohydrodynamics and electromagnetism that expresses the magnetic field using a discrete set of vector distribution functions $\mathbf{g_i}$. The $\mathbf{g_i}$ were first postulated to evolve according to a vector Boltzmann equation of the form

$$\partial_t \mathbf{g_i} + \xi_\mathbf{i} \cdot \nabla \mathbf{g_i} = -\frac{\mathbf{1}}{\tau}\left(\mathbf{g_i} - \mathbf{g_i^{(0)}}\right),$$

where the ξ_i are a discrete set of velocities. The right hand side relaxes the $\mathbf{g_i}$ towards some specified functions $\mathbf{g_i^{(0)}}$ of the fluid velocity $\mathbf{u}$, and of the macroscopic magnetic field given by $\mathbf{B} = \sum_\mathbf{i} \mathbf{g_i}$. Slowly varying solutions obey the equations of resistive magnetohydrodynamics. This lattice Boltzmann formulation has been used in large-scale (up to 1800^3 resolution) simulations of magnetohydrodynamic turbulence. However, this is only the simplest form of Ohm's law. We may simulate more realistic extended forms of Ohm's law using more complex collision operators. A current-dependent relaxation time yields a current-dependent resistivity $\eta(|\nabla \times \mathbf{B}|)$, as used to model "anomalous" resistivity created by small-scale plasma processes. Using a *hydrodynamic* matrix collision operator that depends upon the magnetic field $\mathbf{B}$, we may simulate Braginskii's magnetohydrodynamics, in which the viscosity for strains parallel to the magnetic field lines is much larger than the viscosity for strains in perpendicular directions. Changing the collision operator again, from the above vector Boltzmann equation we may derive the full set of Maxwell's equations, including the displacement current, and Ohm's law,

$$-\frac{1}{c^2}\partial_t \mathbf{E} + \nabla \times \mathbf{B} = \mu_\mathbf{o} \mathbf{J}, \quad \mathbf{J} = \sigma(\mathbf{E} + \mathbf{u} \times \mathbf{B}).$$

The original lattice Boltzmann scheme was designed to reproduce resistive magnetohydrodynamics in the non-relativistic limit. However, the kinetic formulation requires a system of first order partial differential equations with collision terms. This system coincides with the full set of Maxwell's equations and Ohm's law, so we capture a much wider range of electromagnetic phenomena, including electromagnetic waves.

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