The Uncertainty Principle in the Presence of Quantum Memory

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— One consequence of Heisenberg’s uncertainty principle is that no observer can predict the outcomes of two incompatible measurements performed on a system to arbitrary precision. However, this implication is invalid if the observer possesses a quantum memory, a distinct possibility in light of recent technological advances. Entanglement between the system and the memory is responsible for the breakdown of the uncertainty principle, as illustrated by the EPR paradox. In this work we present an improved uncertainty principle which takes this entanglement into account. By quantifying uncertainty using entropy, we show that the sum of the entropies associated with incompatible measurements must exceed a quantity which depends on the degree of incompatibility and the amount of entanglement between system and memory. Apart from its foundational significance, the uncertainty principle motivated the first proposals for quantum cryptography, though the possibility of an eavesdropper having a quantum memory rules out using the original version to argue that these proposals are secure. The uncertainty relation introduced here alleviates this problem and paves the way for its widespread use in quantum cryptography.