Quantum channel convexity and Birkhoff’s theorem

IAN DURHAM, Saint Anselm College — Birkhoff’s Theorem states that doubly stochastic matrices are convex combinations of permutation matrices. Quantum mechanically these matrices are doubly stochastic channels, i.e. they are completely positive maps preserving both the trace and the identity. We expect these channels to be convex combinations of unitary channels and yet it is known that some channels cannot be written that way. Recent work has suggested that \( n \) copies of a single channel might approximate a mixture (convex combination) of unitaries. It turns out that when approached from a mixture of category and group theory, one can show that unital quantum channels and the associated algebra of their Hilbert space, possess category- and group-like properties. From this it can be shown that \( n(n + 1)/2 \) copies of an invertible unital quantum channel may be approximated by a mixture (convex combination) of unitarily implemented channels. This is a stronger result since it means that the asymptotic limit may be approached in fewer steps. It can also be shown that the properties of the channel are preserved in the asymptotic limit.