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Elasticity and response in nearly isostatic periodic lattices

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The emergence of rigidity, especially when complicated by disorder, is a subtle phenomenon that occurs in a wide variety of systems. Isostatic lattices, such as the d-dimensional hypercubic lattice and the $2d$ kagome lattice with nearest neighbor springs, in which the number of contacts z per particle in d-dimensions is equal to $z_c = 2d$, provide simple models, which inform us about systems like jammed solids, glasses, colloidal suspensions, and foams, for the analytic study of general features of the onset of rigidity. Isostatic lattices are marginally stable and may exhibit a non-extensive number of zero modes that can be removed by the addition of bond-bending forces, negative pressure, or additional springs. We use the coherent potential approximation (CPA) to study the onset of rigidity induced by randomly adding next-nearest-neighbor (NNN) bonds to the square and kagome lattices, and we relate the results to the random packings of frictionless spheres at point J. We identify a characteristic frequency scale ω^* and length scale l^* and show that within the CPA they scale, respectively, as Δz and Δz^{-1} where $\Delta z = z - z_c$. This result, which replicates results near jamming, is a result of strongly nonaffine elastic response at small Δz. We find that the frequency-dependent effective NNN spring constant κ obeys a scaling relation $\kappa(\omega)/\kappa(0) = f(\omega/\omega*)$, where $\kappa(0) \sim (\Delta z)^2$. In the square lattice the shear modulus $G(\omega)$ is equal to $\kappa(\omega)$, whereas in the kagome lattice, the shear modulus G_0 at $\Delta z = 0$ is finite and proportional to the spring constant of nearest-neighbor bonds, and $G(\omega) - G_0 \sim \kappa(\omega)$, Finally, we show that the CPA exhibits strong phonon scattering for $\omega > \omega^*$ indicating a Ioffe-Regel limit for heat transport. This work was supported by NSF DMR 0804900.