Prediction, Retrodiction, and the Amount of Information Stored in the Present

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— We introduce an ambidextrous view of stochastic dynamical systems, comparing their forward-time and reverse-time representations and then integrating them into a single time-symmetric representation. The perspective is useful theoretically, computationally, and conceptually. Mathematically, we prove that the excess entropy—a familiar measure of organization in complex systems—is the mutual information not only between the past and future, but also between the predictive and retrodictive causal states. Practically, we exploit the connection between prediction and retrodiction to directly calculate the excess entropy. Conceptually, these lead one to discover new system measures for stochastic dynamical systems: crypticity (information accessibility) and causal irreversibility. Ultimately, we introduce a time-symmetric representation that unifies all of these quantities, compressing the two directional representations into one. The resulting compression offers a new conception of the amount of information stored in the present.