A first-order phase transition at the random close packing of hard spheres

HERNAN MAKSE, YULIANG JIN, City College of New York — Randomly packing spheres of equal size into a container consistently results in a static configuration with a density of $\sim 64\%$. The ubiquity of random close packing (RCP) rather than the optimal crystalline array at 74\% begs the question of the physical law behind this empirically deduced state. Indeed, there is no signature of any macroscopic quantity with a discontinuity associated with the observed packing limit. Here we show that RCP can be viewed as a manifestation of a thermodynamic singularity, which defines it as the “freezing point” in a first-order phase transition between ordered and disordered packing phases. Despite the athermal nature of granular matter, we show the thermodynamic character of the transition in that it is accompanied by sharp discontinuities in volume and entropy at a critical compactivity, which is the intensive variable that plays the role of temperature in granular matter. Our results predict the experimental conditions necessary for the formation of a jammed crystal by calculating the analogue of the ‘entropy of fusion’. This approach is useful since it maps out-of-equilibrium problems in complex systems onto simpler established frameworks in statistical mechanics.