Affine Maps of the Polarization Vector for Quantum Systems of
Arbitrary Dimension

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The operator-sum decomposition (OS) of a mapping from one density matrix to
another has many applications in quantum information science. To this mapping
there corresponds an affine map which provides a geometric description of the density
matrix in terms of the polarization vector representation. This has been thoroughly
explored for qubits since the components of the polarization vector are measurable
quantities (corresponding to expectation values of Hermitian operators) and also
because it enables the description of map domains geometrically. Here we extend
the OS-affine map correspondence to qudits, briefly discuss general properties of the
map, the form for particular important cases, and provide several explicit results
for qutrit maps. We use the affine map and a singular-value-like decomposition,
to find positivity constraints that provide a symmetry for small polarization vector
magnitudes (states which are closer to the maximally mixed state) which is broken
as the polarization vector increases in magnitude (a state becomes more pure). The
dependence of this symmetry on the magnitude of the polarization vector implies
the polar decomposition of the map can not be used as it can for the qubit case.
However, it still leads us to a connection between positivity and purity for general
d-state systems.

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