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### Identifying Topological Order from the Entanglement Spectrum<sup>1</sup>

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The Schmidt decomposition reveals bipartite entanglement of a quantum state. Calculation of the entanglement entropy reduces it to a single number, which can be studied as a function of the size and shape of the entangled regions. However, this reduction discards additional information contained in the full spectrum of the entanglement, which can be presented as a set of (dimensionless) “pseudo-energy” levels spectrum, labeled by quantum numbers such as momentum parallel to the  $(d - 1)$ -dimensional boundary along which the bipartite decomposition of a  $d$ -dimensional system is made. The nature of the entanglement is revealed by this spectrum, much as the elementary excitations and collective modes characterizes condensed-matter states. (The von Neumann entropy is equivalent to the the thermodynamic entropy of the system of pseudo-energy levels at a particular fictitious “temperature”  $k_B T = 1$ .) The previously-unrecognized importance of the spectrum (as opposed to just its entropy) became immediately apparent when the entanglement spectrum of a 2D fractional Quantum Hall state along a 1D cut was first plotted [1]. The gapless spectrum of the conformal field theory related to the topological order of the FQHE can be recognized, and is the only spectrum in model states like the Laughlin or Moore-Read wavefunctions related to cft. For realistic states, corrections due to collective-mode fluctuations give rise to high-pseudo-energy modes that are separated from the gapless (topological) modes by a finite gap. Previously, it had been believed that the extensive  $O(L)$  (“area law”) part of the entanglement entropy of this spectrum was non-universal, and topological order could only be recognized from the  $O(1)$  subleading behavior as the length  $L$  of the cut was scaled. However, while the “pseudo-energy” distribution appears to be non-universal, the distribution of the spectrum *as a function of (true) momentum* does not have this drawback, showing that the topological contribution to the  $O(L)$  part has a universal character not visible in the numerical value of the entropy itself. In general, (including also systems such as topological or Chern insulators [2]), the signature of topological order is the occurrence of gapless mode in the entanglement spectrum, providing a fingerprint from which this order can be identified.

[1] Hui Li and F. D. M. Haldane, Phys. Rev. Lett. **101** 246806 (2008).

[2] F. D. M. Haldane, BAPS.2009.MAR.T13.13.

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