Supersymmetric Structure of two Families of Solitons

ANDREW KOLLER, MAXIM OLSHANII, University of Massachusetts Boston — Solitons have generated considerable interest in the cold atoms and condensed matter communities. We demonstrate that two families of $n$-soliton solutions (with $n$ an integer) — one for the attractive nonlinear Schrödinger (NLS) equation, and one for the sine-Gordon (sG) equation — originate from a quantum-mechanical supersymmetric (QM-SUSY) chain connecting a set of reflectionless operators $\hat{H}_n$. The families consist of breather-type solitons for NLS and multi-(anti)kink solitons with specific velocities for sG. The operators $\hat{H}_n$, which we refer to as Akulin’s Hamiltonians, form reflectionless direct-scattering initial conditions for the inverse scattering method. Such a QM-SUSY chain is analogous to the known connection between QM-SUSY chains of Pöschl-Teller potentials and solitons of the Korteweg-de Vries (KdV) equation. The existence of QM-SUSY chains connecting soliton solutions, now for three different integrable nonlinear equations, sheds light on the underlying mechanisms responsible for soliton generation.

1supported by NSF and ONR